Vamos recordar pontos importantes para se conhecer a vazão de escoamento.

1. Através de uma placa de orifício e conhecendo as curvas características.
   a. Segundo o livro do professor Franco Brunetti

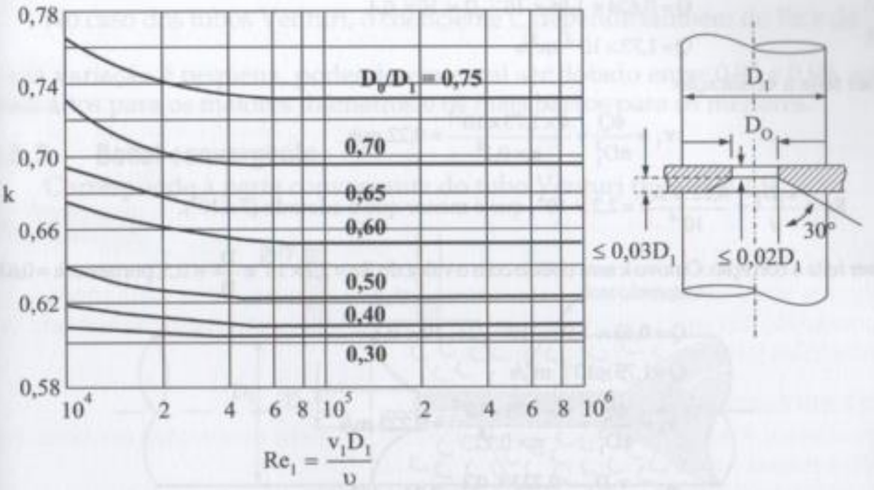
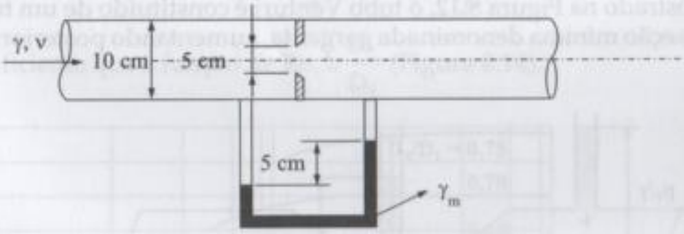


Figura 8.11

EXEMPLO

No orifício da figura, está instalado um manômetro diferencial cujo fluido manométrico, de $\gamma_m = 3 \times 10^4\ N/m^3$, indica um desnível de 5 cm. Sendo o diâmetro do tubo 10 cm e o do orifício 5 cm, determinar a vazão, sabendo que o fluido que escoa é água. ($\gamma = 10^4\ N/m^3$; $\nu = 10^{-6}\ m^2/s$).



Solução

$$A_o = \frac{\pi D_o^2}{4} = \frac{\pi \times 5^2}{4} \times 10^{-4} = 1{,}96 \times 10^{-3}\ m^2$$

Aplicando a equação manométrica:

$$p_1 + \gamma h - \gamma_m h = p_2$$

ou

$$\frac{p_1 - p_2}{\gamma} = \left(\frac{\gamma_m}{\gamma} - 1\right) h$$

portanto:

$$\frac{p_1 - p_2}{\gamma} = \left(\frac{3 \times 10^4}{10^4} - 1\right) \times 0{,}05 = 0{,}1\ m$$

O valor de k deve ser obtido da Figura 8.11. Como, porém, não se conhece a vazão, v_1 não é conhecido, logo não se pode calcular Re. Note-se, porém, que o k, a partir de um certo valor de Re, torna-se constante. O valor de k será adotado e deverá ser verificado posteriormente.

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Logo: $Q = 0{,}624 \times 1{,}96 \times 10^{-3}\sqrt{2 \times 10 \times 0{,}1}$

$Q = 1{,}73 \times 10^{-3}\ m^3/s$

Agora deverá ser feita a verificação:

$$v_1 = \frac{4Q}{\pi D_1^2} = \frac{4 \times 1{,}73 \times 10^{-3}}{\pi \times 0{,}1^2} = 0{,}22\ m/s$$

e $Re = \frac{v_1 D_1}{\nu} = \frac{0{,}22 \times 0{,}1}{10^{-6}} = 2{,}2 \times 10^4$, que é menor que o adotado (7×10^4).

Portanto, deve ser feita a correção. O novo k será obtido com o valor do $Re = 2{,}2 \times 10^4$ e $\frac{D_o}{D_1} = 0{,}5$; portanto: $k = 0{,}63$.

Logo: $Q = 0{,}63 \times 1{,}96 \times 10^{-3}\sqrt{2 \times 10 \times 0{,}1}$

$Q = 1{,}75 \times 10^{-3}\ m^3/s$

Verificação: $$v_1 = \frac{4Q}{\pi D_1^2} = \frac{4 \times 1{,}75 \times 10^{-3}}{\pi \times 0{,}1^2} = 0{,}223\ m/s$$

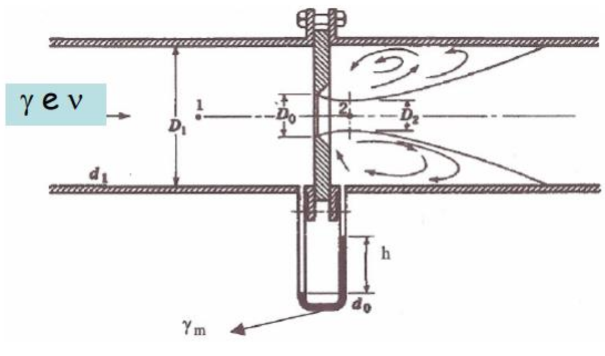
$$Re = \frac{v_1 D_1}{\nu} = \frac{0{,}223 \times 0{,}1}{10^{-6}} = 2{,}23 \times 10^4$$

Note-se que o Re variou muito pouco; logo, a vazão obtida na segunda tentativa pode ser adotada como verdadeira. Assim: Q = 1,75 L/s.

Importante observar que existem divergências nos gráficos apresentados e isto justificaria uma pesquisa inicialmente bibliográfica.

- Segundo meu material publicado no site www.escoladavida.eng.br

As placas de orifício semelhantes a da figura apresentam as curvas universais representadas no slide seguinte. Através do desenvolvimento teórico, onde considerou-se o fluido ideal, obteve-se a equação: $Q = 0{,}01248 \cdot h^{0,5}$ (com h em metro e Q em m^3/s). Pede-se determinar a vazão da instalação e o peso específico do fluido manométrico (γ_m) para a situação considerada.



Dados

$D_1 = 50\ mm$; $D_0 = 31{,}63\ mm$;

$h = 40\ cm$; $g = 9{,}8\ \frac{m}{s^2}$;

$\gamma = 9800\ \frac{N}{m^3}$ e $\nu = 10^{-6}\ \frac{m^2}{s}$

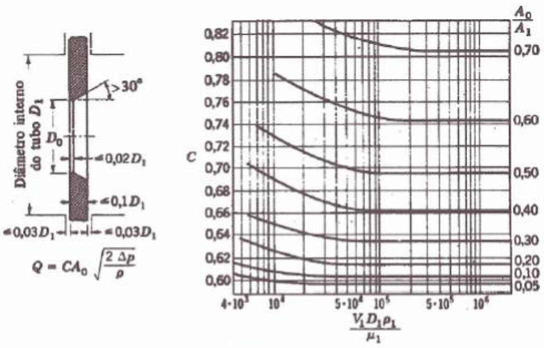


Figura 5.24

O coeficiente de escoamento (C) é definido pela equação 5.29.

$$C = \frac{C_d}{\sqrt{1 - C_C^2 \cdot \left(\frac{A_0}{A_1}\right)^2}} \qquad \text{equação 5.29}$$

Respostas:

$$Q \cong 5{,}21 \times 10^{-3}\ \frac{m^3}{s}$$

$$\gamma_m \cong 135959{,}82\ \frac{N}{m^3}$$

Figura extraída do sítio:
http://www.escoladavida.eng.br/mecflubasica/aula4_unidade5.htm

- Segundo o livro de Introdução à mecânica dos fluidos de Robert W. Fox e outros

A Placa de Orifício

A placa de orifício (Fig. 8.19) é uma placa fina que pode ser interposta entre flanges de tubos. Devido a sua geometria simples, é de baixo custo e de fácil instalação ou reposição. A borda viva do orifício não deve ficar incrustada com depósitos ou matéria em suspensão. Contudo, material em suspensão pode se acumular no lado da entrada de um orifício concêntrico em um tubo horizontal; uma placa de orifício excêntrico posicionado rente ao fundo do tubo pode ser instalada para evitar esse problema. As principais desvantagens do orifício são a sua capacidade limitada e a elevada perda de carga permanente decorrente da expansão não-controlada a jusante do elemento medidor.

As tomadas de pressão para orifícios podem ser colocadas em diversos locais, conforme mostrado na Fig. 8.19 (consulte [10] ou [26] ou Normas da ABNT como a NBR ISO 5167 para mais detalhes).

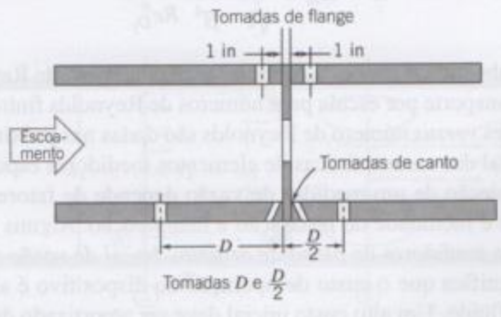


Fig. 8.19 Geometria de orifício e localização de tomadas de pressão [10].

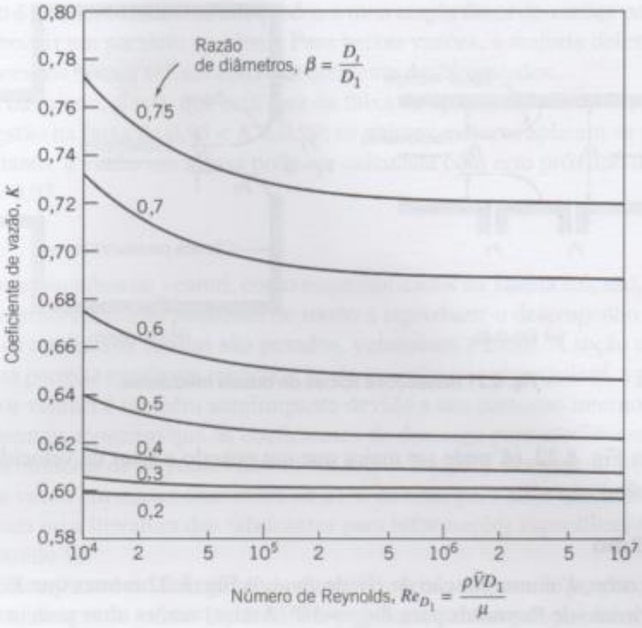


Fig. 8.20 Coeficientes de vazão para orifícios concêntricos com tomadas de canto.

Como a localização das tomadas de pressão influencia o coeficiente de vazão empírico, valores para C ou K consistentes com a localização das tomadas devem ser selecionados de manuais ou de normas preferencialmente.

A equação de correlação recomendada para um orifício concêntrico com tomadas de canto [10] é

$$C = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^{8} + \frac{91.71\beta^{2.5}}{Re_{D_1}^{0.75}} \tag{8.59}$$

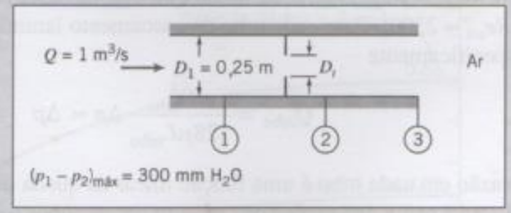
A Eq. 8.59 prediz os coeficientes de descarga com precisão de ±0,6% para $0,2 < \beta < 0,75$ e para $10^4 < Re_{D_1} < 10^7$. Alguns coeficientes de vazão calculados com as Eqs. 8.59 e 8.55 são apresentados na Fig. 8.20.

Uma equação de correlação similar é disponível para placas de orifício com tomadas de pressão D e $D/2$ (posicionadas a um diâmetro de tubo a montante e meio diâmetro de tubo a jusante da placa). As tomadas de flange requerem uma correlação diferente para cada diâmetro de tubo. As tomadas de pressão localizadas a $2\frac{1}{2}D$ e $8D$ não são mais recomendadas para trabalhos de precisão.

O Problema-Exemplo 8.12, descrito mais adiante nesta seção, ilustra a aplicação de dados do coeficiente de vazão no dimensionamento de uma placa de orifício.

PROBLEMA-EXEMPLO 8.12

DADOS: Escoamento através de um duto com placa de orifício, conforme mostrado.



DETERMINAR: **(a)** D_t.
(b) A perda de carga entre as seções ① e ③.
(c) O grau de concordância com os dados da Fig. 8.23.

SOLUÇÃO:
A placa de orifício pode ser projetada usando a Eq. 8.56 e os dados da Fig. 8.20.

Equação básica: $$\dot{m}_{real} = KA_t\sqrt{2\rho(p_1 - p_2)} \qquad (8.56)$$

onsiderações: (1) Escoamento permanente.
(2) Escoamento incompressível.

omo $A_t/A_1 = (D_t/D_1)^2 = \beta^2$,

$$\dot{m}_{real} = K\beta^2 A_1\sqrt{2\rho(p_1 - p_2)}$$

$$K\beta^2 = \frac{\dot{m}_{real}}{A_1\sqrt{2\rho(p_1 - p_2)}} = \frac{\rho Q}{A_1\sqrt{2\rho(p_1 - p_2)}} = \frac{Q}{A_1}\sqrt{\frac{\rho}{2(p_1 - p_2)}}$$

$$= \frac{Q}{A_1}\sqrt{\frac{\rho}{2g\rho_{H_2O}\Delta h}}$$

$$= \frac{1\ m^3}{s} \times \frac{4}{\pi}\frac{1}{(0{,}25)^2\ m^2}\left[\frac{1}{2} \times 1{,}23\frac{kg}{m^3} \times \frac{s^2}{9{,}81\ m} \times \frac{m^3}{999\ kg} \times \frac{1}{0{,}30\ m}\right]^{1/2}$$

$$K\beta^2 = 0{,}295 \quad \text{ou} \quad K = \frac{0{,}295}{\beta^2} \qquad (1)$$

omo K é uma função de β (Eq. 1) e de Re_{D_1} (Fig. 8.20), devemos promover iterações para determinar β. O número de eynolds no duto é

$$Re_{D_1} = \frac{\rho\bar{V}_1 D_1}{\mu} = \frac{\rho(Q/A_1)D_1}{\mu} = \frac{4Q}{\pi\nu D_1}$$

$$Re_{D_1} = \frac{4}{\pi} \times \frac{1\ m^3}{s} \times \frac{s}{1{,}46 \times 10^{-5}\ m^2} \times \frac{1}{0{,}25\ m} = 3{,}49 \times 10^5$$

açamos $\beta = 0{,}75$. Da Fig. 8.20, K deve ser 0,72. Da Eq. 1,

$$K = \frac{0{,}295}{(0{,}75)^2} = 0{,}524$$

ssim, nossa estimativa para β é grande demais. Façamos $\beta = 0{,}70$. Da Fig. 8.20, K deve ser 0,69. Da Eq. 1,

$$K = \frac{0{,}295}{(0{,}70)^2} = 0{,}602$$

ssim, nossa estimativa para β ainda é grande demais. Façamos $\beta = 0,65$. Da Fig. 8.20, K deve ser 0,67. Da Eq. 1,

$$K = \frac{0,295}{(0,65)^2} = 0,698$$

xiste concordância satisfatória com $\beta \simeq 0,66$ e

$$D_t = \beta D_1 = 0,66(0,25 \text{ m}) = 0,165 \text{ m} \qquad D_t$$

ara avaliar a perda de carga permanente para este dispositivo, nós poderíamos simplesmente usar a razão de diâmetros $\beta \approx$,66 na Fig. 8.23; mas, em vez disso, faremos a determinação a partir dos dados disponíveis. Para avaliar a perda de carga ermanente, aplique a Eq. 8.29 entre as seções ① e ③.

quação básica:

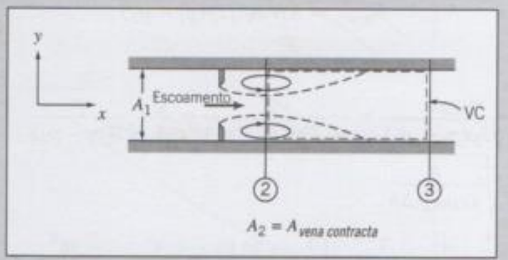
$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_3}{\rho} + \alpha_3 \frac{\bar{V}_3^2}{2} + gz_3\right) = h_{l_T} \qquad (8.29)$$

onsiderações: (3) $\alpha_1 \bar{V}_1^2 = \alpha_3 \bar{V}_3^2$
(4) Δz desprezível.

ntão,

$$h_{l_T} = \frac{p_1 - p_3}{\rho} = \frac{p_1 - p_2 - (p_3 - p_2)}{\rho} \qquad (2)$$

A Eq. 2 é a nossa aproximação: Nós encontraremos $p_1 - p_3$ fazendo $p_1 - p_2 = 300$ mmH_2O (máxima pressão diferencial pe mitida na placa) e obtendo um valor para $p_3 - p_2$ a partir da componente x da equação da quantidade de movimento para u volume de controle entre as seções ② e ③.



Equação básica:

$$= 0(5) \quad = 0(1)$$

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{VC} u \rho d\Psi + \int_{SC} u \rho \vec{V} \cdot d\vec{A} \qquad (4.18$$

Considerações: (5) $F_{B_x} = 0$.
(6) Escoamento uniforme nas seções ② e ③.
(7) Pressão uniforme através do duto nas seções ② e ③.
(8) Força de atrito desprezível sobre o VC.

Assim, simplificando e rearranjando,

$$(p_2 - p_3)A_1 = u_2(-\rho\hat{V}_2A_2) + u_3(\rho\hat{V}_3A_3) = (u_3 - u_2)\rho Q = (\hat{V}_3 - \hat{V}_2)\rho Q$$

ou

$$p_3 - p_2 = (\hat{V}_2 - \hat{V}_3)\frac{\rho Q}{A_1}$$

Mas $\hat{V}_3 = Q/A_1$, e

$$\hat{V}_2 = \frac{Q}{A_2} = \frac{Q}{0{,}65A_t} = \frac{Q}{0{,}65\beta^2 A_1}$$

Então,

$$p_3 - p_2 = \frac{\rho Q^2}{A_1^2}\left[\frac{1}{0.65\beta^2} - 1\right]$$

$$p_3 - p_2 = 1{,}23\,\frac{\text{kg}}{\text{m}^3} \times \frac{(1)^2\,\text{m}^6}{\text{s}^2} \times \frac{4^2}{\pi^2}\frac{1}{(0{,}25)^4\,\text{m}^4}\left[\frac{1}{0{,}65(0{,}66)^2} - 1\right]\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$p_3 - p_2 = 1290\ \text{N/m}^2$$

A razão de diâmetros, β, foi selecionada para dar deflexão máxima no manômetro na vazão máxima. Por conseguinte,

$$p_1 - p_2 = \rho_{H_2O}g\Delta h = 999\,\frac{\text{kg}}{\text{m}^3} \times 9{,}81\,\frac{\text{m}}{\text{s}^2} \times 0{,}30\ \text{m} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 2940\ \text{N/m}^2$$

Substituindo na Eq. 2, resulta em

$$h_{l_T} = \frac{p_1 - p_3}{\rho} = \frac{p_1 - p_2 - (p_3 - p_2)}{\rho}$$

$$h_{l_T} = (2940 - 1290)\,\frac{\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{1{,}23\ \text{kg}} = 1340\ \text{N}\cdot\text{m/kg}$$

- Conteúdo extraído do livro: “ENGINEERING FLID MECHANICS” escrito por ALAN MIRONER – McGRAW-HILL

7.4 FLOWMETERS

Pipeline flow rates are metered by placing a constriction in the line and measuring the pressure change across it. The *venturi* shown in Fig. 7.26 is one of the most common flowmeters using this principle. A U-tube manometer measures the pressure drop between the throat, or minimum area, of the venturi and the line. A venturi is a combination of a nozzle and diffuser, and the remarks about nozzle and diffuser contours made in Secs. 5.2 and 7.3 also apply to the venturi.

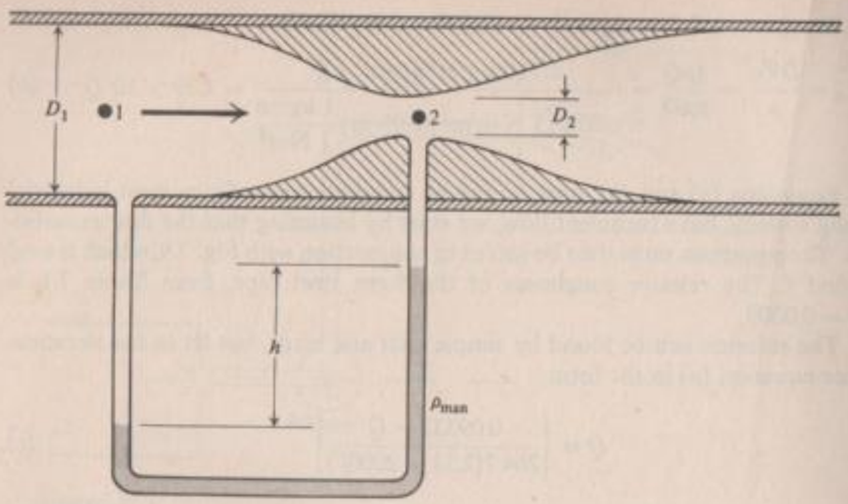


Figure 7.26 Venturi flowmeter.

To calculate the ideal flow rate through the venturi, we apply the ideal Bernoulli equation between locations 1 and 2. The result is the same as Eq. (5.20). We use the subscript *i* for ideal, and the ideal throat velocity is then

$$V_{2_i} = \sqrt{\frac{2}{\rho}\frac{p_1 - p_2}{1 - (D_2/D_1)^4}} \tag{7.38}$$

The ideal flow rate passed by the venturi is then

$$\dot{Q}_i = A_2 V_{2_i} = \frac{\pi}{4} D_2^2 \sqrt{\frac{2}{\rho}\frac{p_1 - p_2}{1 - (D_2/D_1)^4}} \tag{7.39}$$

The pressure difference registered by the manometer is

$$p_1 - p_2 = \left(\frac{\rho_{man}}{\rho} - 1\right)\rho g h$$

Then

$$\dot{Q}_i = \frac{\pi}{4} D_2^2 \sqrt{\frac{2[(\rho_{man}/\rho) - 1]}{1 - (D_2/D_1)^4} g h} \tag{7.40}$$

The flow rate in the pipe is directly related to the manometer column-height difference. Note that the equations developed in this section are for an incompressible flow. As the flow velocity increases and the fluid becomes increasingly compressible, the relationship between pressure, velocity, and area changes dramatically. Thus, the equations developed are valid only for liquid flows and low-speed gas flows.

In the actual flow, frictional shear and turbulence cause a loss of mechanical energy and a reduction in the flow rate. An experimentally measured coefficient, called the *discharge coefficient* C_d is used to relate the actual flow rate to the ideal flow rate

$$C_d = \frac{\dot{Q}_a}{\dot{Q}_i} \tag{7.41}$$

Figure 7.27 shows the experimentally determined correlation of discharge coefficient with pipe Reynolds number R_{D_1} for a large number of venturis having pipe diameters D_1 of 2 in and larger. The venturis had throat-to-pipe-diameter ratios (D_2/D_1) in the range 0.30 to 0.75. Above $\mathrm{R}_{D_1} = 2 \times 10^5$, the discharge coefficient is constant at a value of 0.984. The converging-diverging contour guides the flow with a minimum of turbulence and mechanical-energy loss.

Venturis are expensive to fabricate, and often the simpler and cheaper flow nozzle shown in Fig. 7.28 is used. The purpose of the converging contour is to guide the flow smoothly so that the streamlines at the nozzle exit are parallel. After leaving the nozzle, the flow spreads in an unguided manner to fill the pipe. This free or uncontrolled spreading of the streamlines results in turbulence and a mechanical-energy loss.

Figure 7.29 shows a representative plot of the experimentally determined discharge coefficient for a nozzle as a function of throat Reynolds number R_{D_2}. All diameter ratios D_2/D_1 between 0.15 and 0.75 are contained between the two curves. The discharge coefficient is applied to the ideal nozzle flow rate, which is also calculated from Eqs. (7.39) or (7.40).

The discharge coefficient also varies for different nozzle contours, pipe sizes, and pressure-tap locations. Figure 7.29 is a correlation for ASME long-radius flow

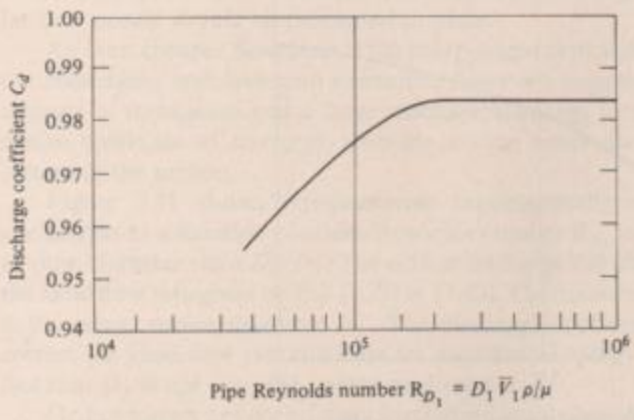


Figure 7.27 Venturi discharge coefficient. *(Adapted from "Fluid Meters: Their Theory and Application," Report of ASME Research Committee on Fluid Meters, 5th ed., American Society of Mechanical Engineers, New York, 1959.)*

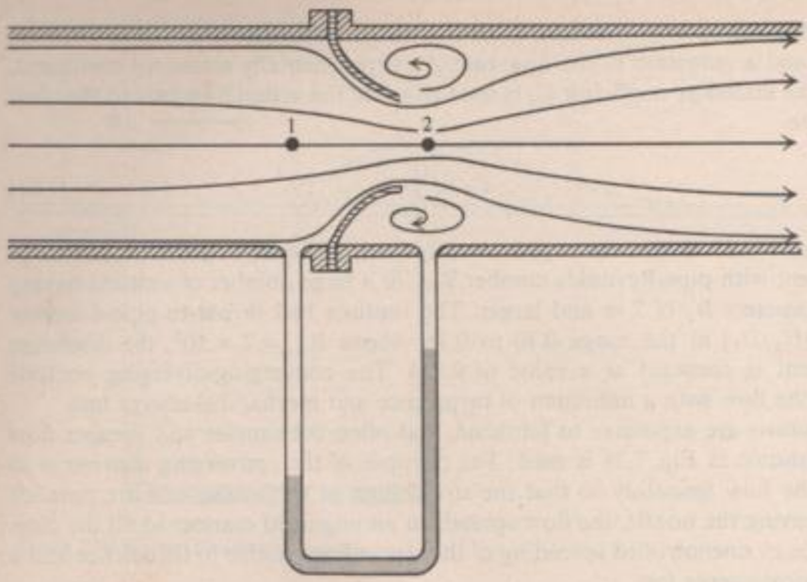


Figure 7.28 Flow nozzle.

nozzles in 2-in pipes. The upstream pressure tap is located a distance D_1 upstream from the beginning of the nozzle, and the downstream pressure tap is located a distance $\frac{1}{2}D_1$ downstream from the beginning of the nozzle.

When Fig. 7.29 is used to predict the flow rate in other nozzles, its accuracy is seriously impaired if the leaving flow does not have parallel streamlines. The flow

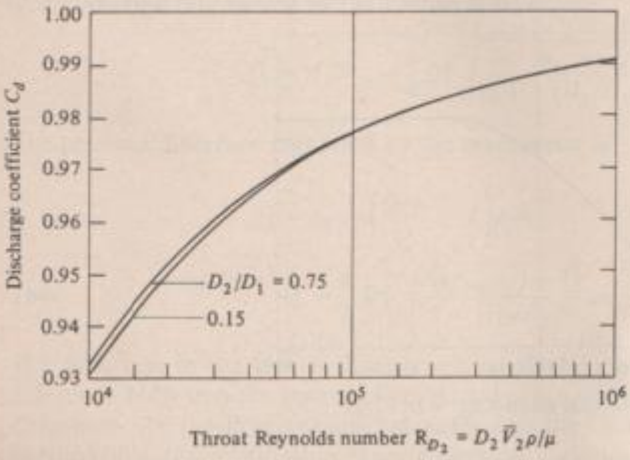


Figure 7.29 Nozzle discharge coefficient. *(Data from Howard S. Bean (ed.), "Fluid Meters: Their Theory and Application," 6th ed., American Society of Mechanical Engineers, New York, 1971.)*

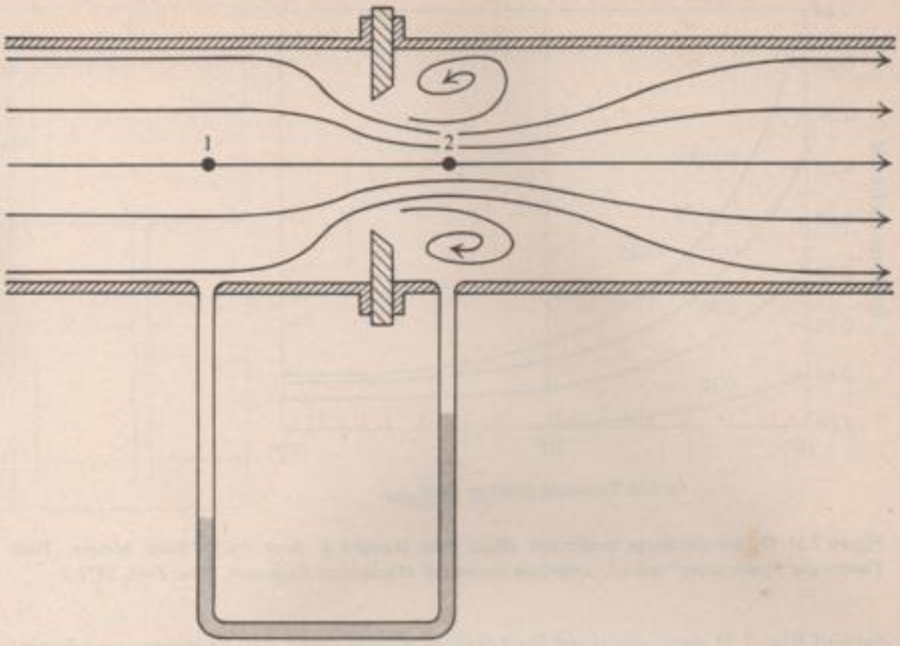


Figure 7.30 Sharp-edged orifice.

continues to converge, reaching a minimum area, or vena contracta, downstream of the nozzle exit. The diameter of the vena contracta is less than that of the nozzle exit, resulting in an error in the predicted flow rate. For high accuracy, the particular flow nozzle should be calibrated in place.

An even cheaper flowmeter is the sharp-edged orifice shown in Fig. 7.30. Both the converging and diverging streamline flows are unguided, resulting in a large amount of turbulence and a large mechanical-energy loss. The flow leaving the orifice continues to converge, reaching a vena contracta some distance downstream of the orifice.

Figure 7.31 shows representative experimentally measured orifice flow coefficients as a function of orifice Reynolds number R_{D_2} for parameters of orifice-to-pipe-diameter ratio D_2/D_1. The orifice discharge coefficient is also applied to the ideal flow rate given by Eq. (7.39) or (7.40). The diameter used in the equations is the actual orifice diameter D_2. The discharge coefficients given in Fig. 7.31 correct the ideal flow rate not only for mechanical-energy losses but also for the fact that D_2 is not the vena contracta diameter.

Orifice meters are notoriously inaccurate because of the difficulty in positioning the downstream pressure tap precisely at the vena contracta. To alleviate this difficulty the discharge coefficient is often determined for a fixed location of the pressure taps without regard for the actual location of the vena contracta. The

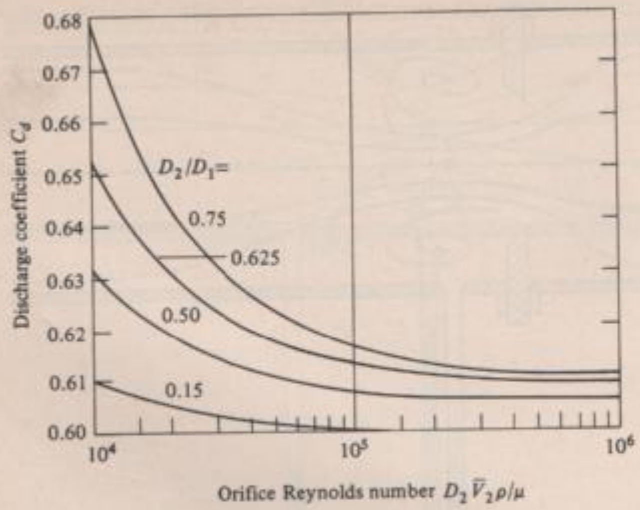
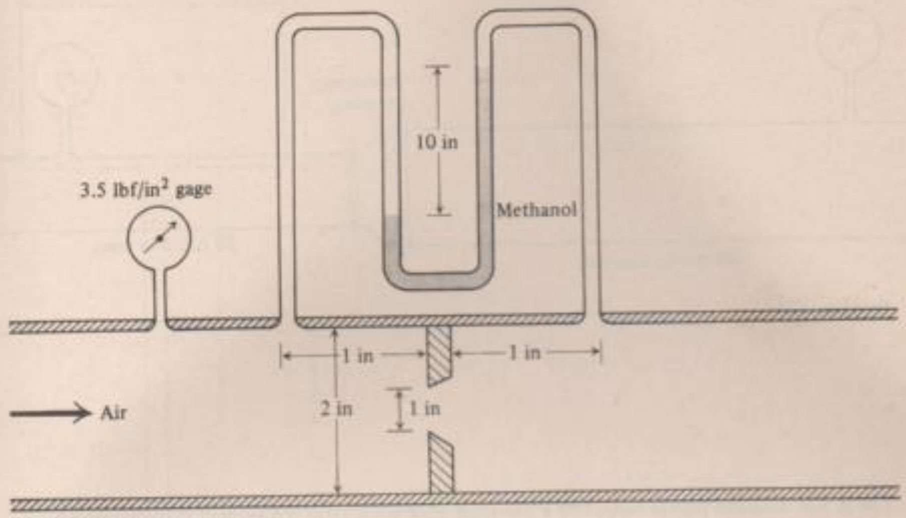


Figure 7.31 Orifice discharge coefficient. *(Data from Howard S. Bean (ed.), "Fluid Meters: Their Theory and Application," 6th ed., American Society of Mechanical Engineers, New York, 1971.)*

data of Fig. 7.31 were obtained for pressure taps located 1 in upstream and downstream of the orifice plate. The discharge coefficient also depends slightly on the size of the pipe. The data of Fig. 7.31 are for an orifice mounted in a 2-in pipe. For any degree of accuracy the orifice meter must be calibrated in place.

PROBLEMS

7.17 A 1-in-diameter sharp-edged orifice is mounted in a 2-in-diameter pipe. The pipe is carrying air at 80°F. The upstream pressure gage indicates a pressure of 3.5 lbf/in², and the column difference of the methanol manometer is 10 in. Estimate the flow rate of air through the pipe.



Problem 7.17

7.18 A well-designed nozzle of area ratio 4 is used to meter the flow rate through a pipe. Estimate the loss coefficient for this nozzle flow based on the pipe kinetic-energy head.

7.19 The flow through an orifice having an area ratio of 4 has a vena contracta area equal to six-tenths of the orifice-hole area. Estimate the loss coefficient for this orifice flow based on the pipe kinetic-energy head.

7.20 A venturi with area ratio 4 is used to meter a flow. The converging portion of the venturi is a well-designed nozzle having a velocity coefficient of 0.98. The diverging portion of the venturi is a conical diffuser having a half angle of 5°. Estimate the loss coefficient based on the pipe kinetic-energy head.

7.21 Using your knowledge of a flow restriction which accelerates a flow to a vena contracta and then allows the flow to undergo an uncontrolled deceleration, discuss the high loss coefficients associated with flow through valves. Use equations where possible to support your discussion.

7.22 Reconsider Example 5.2 with a frictional mechanical-energy loss for the flow over the ramp. For the same approach conditions, h_1 and V_1, discuss the effect of friction on the downstream water depth h_2. Use equations to support your discussion.

- Conteúdo extraído do livro: "Engineering Fluid Mechanics" escrito por Roberson/Crowe e editado por Houghton Mifflin Company – International Dolphin Edition

Orifice

A restricted opening through which fluid flows is an *orifice,* and if the geometric characteristics of the orifice plus the properties of the fluid are known, then the orifice can be used to measure flow rates. Consider flow through the sharp-edged pipe orifice shown in Fig. 13-10. It is seen that the streamlines continue to converge

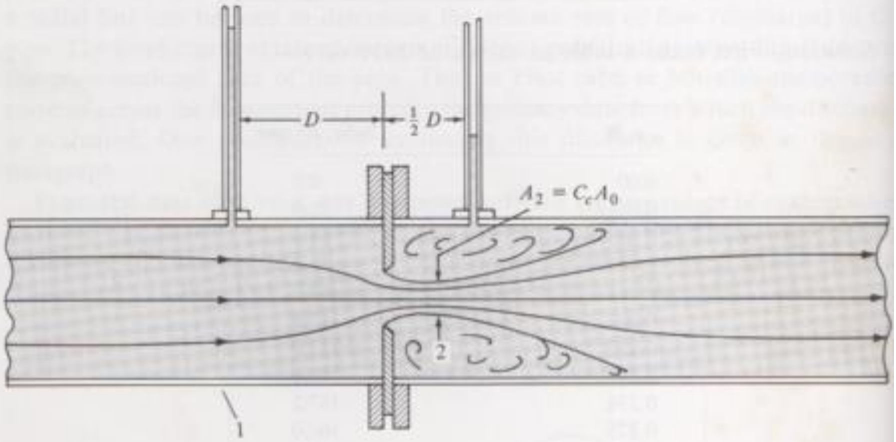


FIGURE 13-10 Flow through a pipe orifice.

a short distance downstream of the plane of the orifice; hence, the minimum-flow area is actually smaller than the area of the orifice. To relate the minimum-flow area, often called the contracted area of the jet or *vena contracta,* to the area of the orifice A_o, we use the contraction coefficient, which is defined as

$$A_j = C_c A_o$$

$$C_c = \frac{A_j}{A_o}$$

Then for a circular orifice,

$$C_c = \frac{(\pi/4)d_j^2}{(\pi/4)d^2} = \left(\frac{d_j}{d}\right)^2$$

Because d_j and d_2 are identical, we also have $C_c = (d_2/d)^2$. At low values of the Reynolds number, C_c is a function of the Reynolds number; however, at high values of the Reynolds number, C_c is only a function of the geometry of the orifice. For d/D ratios less than 0.3, C_c has a value of approximately 0.62; however, as d/D is increased to 0.8, C_c increases to a value of 0.72. The derivation of the discharge equation for the orifice is started by writing the Bernoulli equation between section 1 and section 2 in Fig. 13-10. We then have

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Now, V_1 is eliminated by means of the continuity equation $V_1A_1 = V_2A_2$; and solving for V_2 gives

$$V_2 = \left\{\frac{2g[(p_1/\gamma + z_1) - (p_2/\gamma + z_2)]}{1 - (A_2/A_1)^2}\right\}^{1/2} \tag{13-1a}$$

However, $A_2 = C_cA_o$ and $h = p/\gamma + z$; so that Eq. (13-1*a*) reduces to

$$V_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - C_c^2A_o^2/A_1^2}} \tag{13-1b}$$

Our primary objective is to obtain an expression for discharge in terms of the h_1, h_2 and the geometric characteristics of the orifice. The discharge is given by V_2A_2; hence, when we multiply both sides of Eq. (13-1*b*) by $A_2 = C_cA_o$ we obtain the desired result:

$$Q = \frac{C_cA_o}{\sqrt{1 - C_c^2A_o^2/A_1^2}}\sqrt{2g(h_1 - h_2)} \tag{13-2}$$

Equation (13-2) is the discharge equation for the flow of an incompressible inviscid fluid through an orifice. However, this is only valid at relatively high Reynolds numbers. For low and moderate values of the Reynolds number, viscous effects are significant and an additional coefficient must be applied to the discharge equation to relate the ideal to the actual flow. This is called the *coefficient of velocity* C_v; thus, for viscous flow in an orifice we have the following discharge equation:

$$Q = \frac{C_vC_cA_o}{\sqrt{1 - C_c^2A_o^2/A_1^2}}\sqrt{2g(h_1 - h_2)}$$

The product C_vC_c is called the *discharge coefficient* C_d, and the combination $C_vC_c/(1 - C_c^2A_o^2/A^2)^{1/2}$ is called the *flow coefficient* K. Thus, we have $Q = KA_o\sqrt{2g(h_1 - h_2)}$, where

$$K = \frac{C_d}{\sqrt{1 - C_c^2A_o^2/A_1^2}}$$

If Δh is defined as $h_1 - h_2$, then the final form of the discharge equation for an orifice reduces to

$$Q = KA_o\sqrt{2g\,\Delta h} \tag{13-3}$$

Experimentally determined values of K as a function of d/D and Reynolds number

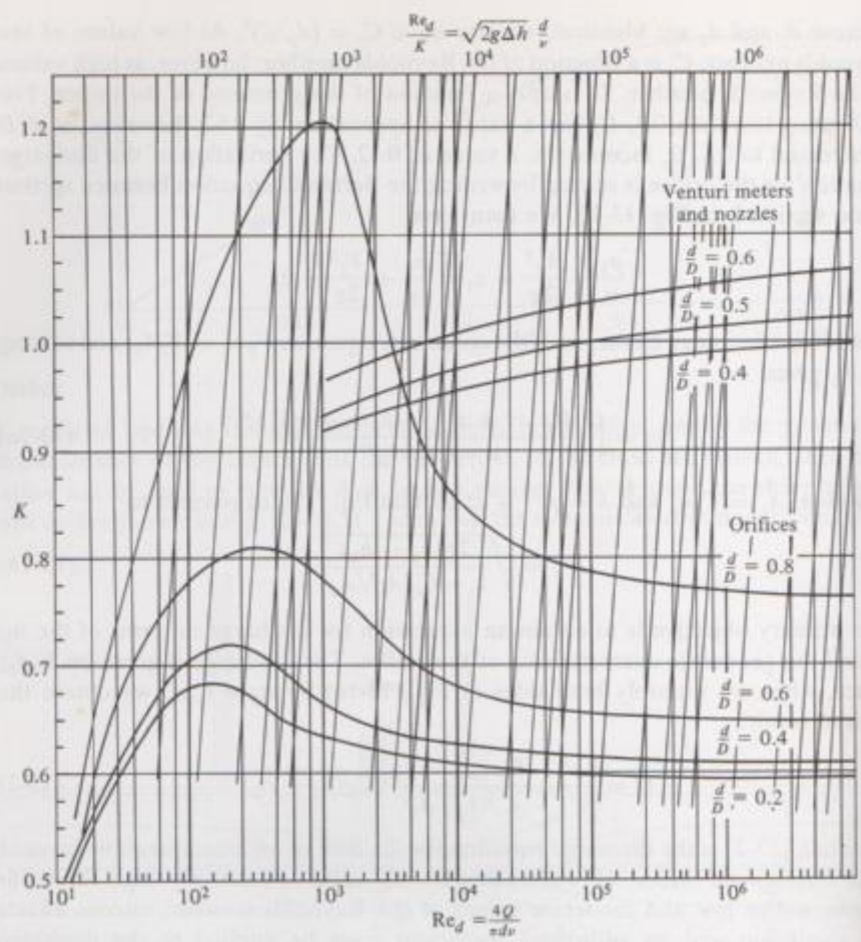


FIGURE 13-11 Flow coefficient K and Re_d/K vs. the Reynolds number for orifices, nozzles, and Venturi meters. [After Johansen (4) and ASME (1).]

based upon orifice size, $4Q/\pi\, d\nu$, are given in Fig. 13-11. If Q is given, Re_d is equal to $4Q/\pi\, d\nu$. K is obtained from Fig. 13-11 (using the vertical lines and the bottom scale) and Δh is then computed from Eq. (13-3). However, we are often confronted with the problem of determining the discharge Q when a certain value of Δh is given. When Q is to be determined, we do not have a direct way to obtain K by entering Fig. 13-11 with Re, because Re is a function of the flow rate which is still unknown. Hence, another scale which does not involve Q is constructed on the graph of Fig. 13-11. The variables for this scale are obtained in the following manner: because $Re_d = 4Q/\pi\, d\nu$ and $Q = K\pi d^2/4\sqrt{2g\,\Delta h}$, we can write Re_d in terms of Δh as

$$Re_d = K\sqrt{2g\,\Delta h}\,\frac{d}{\nu}$$

or

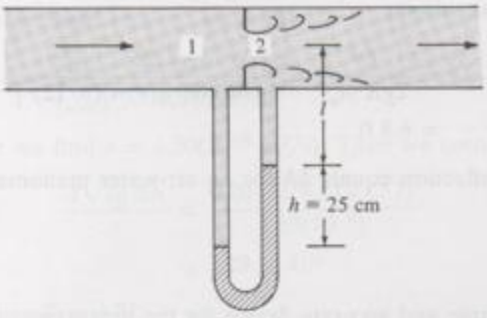
$$\frac{\text{Re}_d}{K} = \sqrt{2g\,\Delta h}\,\frac{d}{\nu}$$

Thus the slanted lines and the top scale in Fig. 13-11 are used when Δh is known and the flow rate is to be determined.

The literature on orifice flow contains numerous discussions concerning the optimum placement of pressure taps on both the upstream and downstream side of the orifice. The data given in Fig. 13-11 are for "corner taps." That is, on the upstream side the pressure readings were taken immediately upstream of the plate orifice (at the corner of the orifice plate and the pipe wall) and the downstream tap was at a similar downstream location. However, it should be noted that pressure data from flange taps (1 in. upstream and 1 in. downstream) and from the taps shown in Fig. 13-10 all yield virtually the same values for K—the differences are no greater than the deviations involved in reading Fig. 13-11. For more precise values of K with specific types of taps, the reader is directed to the ASME report on fluid meters (1).

EXAMPLE 13-2 A 15-cm orifice is located in a horizontal 24-cm water pipe and a water-mercury manometer is connected to either side of the orifice. When the deflection on the manometer is 25 cm, what is the discharge in the system? Assume that the water temperature is 20°C.

Solution The discharge is given by Eq. (13-3): $Q = KA_o\sqrt{2g\,\Delta h}$. To either enter Fig. 13-11 or use Eq. (13-3), we will need to first evaluate Δh, the change in piezometric head in meters of fluid that is flowing. This is obtained by applying the equation of hydrostatics to the manometer shown below.



Writing the manometer equation from point 1 to point 2 we have

$$p_1 + \gamma_w l + \gamma_w h - \gamma_{Hg} h - \gamma_w l = p_2$$

Then,

$$\frac{p_1 - p_2}{\gamma_w} = \Delta h = \frac{h(\gamma_{Hg} - \gamma_w)}{\gamma_w} = h\left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right)$$

For this example,

$$\Delta h = 0.25\text{ m }(13.6 - 1)$$
$$h = 3.15\text{ m of water}$$

The kinematic viscosity of water at 20°C is 1.0×10^{-6} m²/s; so we now can compute

$d\sqrt{2g\,\Delta h}/\nu$, the parameter that is needed to enter Fig. 13-11:

$$\frac{d\sqrt{2g\,\Delta h}}{\nu} = \frac{0.15\text{ m}\sqrt{2(9.81\text{ m/s}^2)(3.15\text{ m})}}{1.0 \times 10^{-6}\text{ m}^2/\text{s}} = 1.2 \times 10^6$$

From Fig. 13-11 with $d/D = 0.625$, we read K to be 0.66 (interpolated). Hence,

$$\begin{aligned} Q &= 0.66\,A_o\sqrt{2g\,\Delta h} \\ &= 0.66\frac{\pi}{4}d^2\sqrt{2(9.81\text{ m/s}^2)(3.15\text{ m})} \\ &= 0.66\,(0.785)(0.15^2\text{ m}^2)(7.87\text{ m/s}) \\ &= 0.092\text{ m}^3/\text{s} \qquad \Leftarrow \text{answer} \end{aligned}$$

EXAMPLE 13-3 An air-water manometer is connected to either side of an 8-in. orifice in a 12-in. water pipe. If the maximum flow rate is 5 cfs, what is the deflection on the manometer? The water temperature is 60°F.

Solution The Reynolds number $Re_d = 4Q/\pi d\nu$ is first computed so that we may enter Fig. 13-11 to obtain the flow coefficient K; then K will be used in Eq. (13-3) to compute the deflection Δh.

$$\begin{aligned} Re_d &= \frac{4Q}{\pi d\nu} = \frac{(4)(5)}{3.14(8/12)(1.22)(10^{-5})} \\ &= 7.8 \times 10^5 \end{aligned}$$

From Fig. 13-11 for $d/D = 8/12 = 0.667$ by interpolating between curves of $d/D = 0.6$ and $d/D = 0.8$, we read K to be approximately 0.68. Then from $Q = KA_o\sqrt{2g\,\Delta h}$ we obtain

$$\begin{aligned} \Delta h &= \frac{Q^2}{2gK^2A_o^{\,2}} = \frac{25}{64.4(0.68^2)[(\pi/4)(8/12)^2]^2} \\ &= 6.8\text{ ft} \qquad \Leftarrow \text{answer} \end{aligned}$$

The manometer deflection equals Δh for an air-water manometer.

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